ALGORITHMS WITH PREDICTIONS

WARMING UP AND RENTING SKIS

Adam Polak

SOFSEM PhD School, Kraków, February 9th, 2026

MOTIVATION

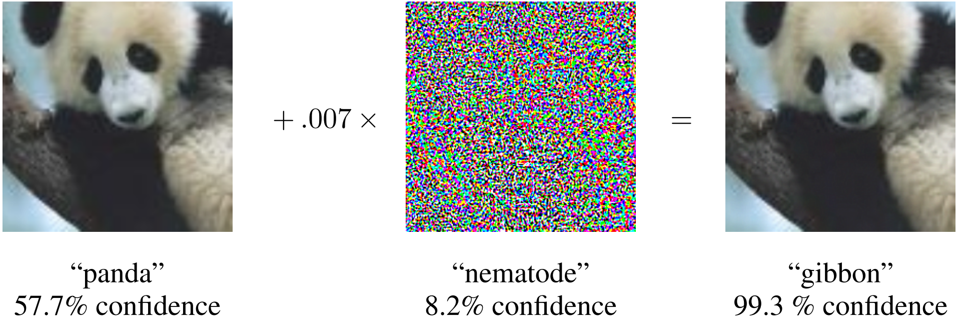
Classical algorithms

- Worst-case guarantees
- Overly pessimistic on easy instances

Machine learning

- Powerful most of the time
- No guarantees, can go crazy

RECALL: ADVERSARIAL EXAMPLES



Source: arxiv.org/abs/1412.6572

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Best of both worlds: learning-augmented algorithms

Input + black-box predictions (e.g., coming from ML model)

ALGORITHMS WITH PREDICTIONS = ALPS

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- **Robustness:** worst-case guarantees, even when predictions adversarial
- **Smoothness:** performance degrades slowly in the prediction error
- **Learnability:** good predictions can be learned efficiently

WARM UP:

SKI RENTAL

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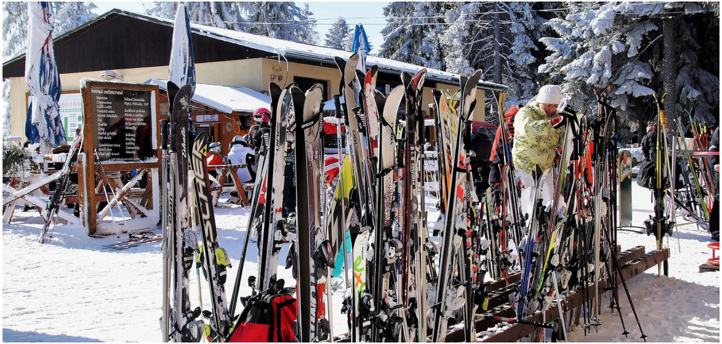

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- rent the skis for one more day paying 1, or
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- predicted length of ski season: y days
- blindly following the prediction: if $y > B$, buy on day 1; if $y \leq B$, never buy
  $\text{cost}(ALG) \leq \text{cost}(OPT) + |x - y|$ (consistent, smooth, but not robust)

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- deterministic algorithm with a trade-off parameter $\lambda \in (0, 1)$
  - if $y > B$, buy on day λB; if $y \leq B$, buy on day B/λ

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  - correct prediction: $(1 + \lambda)$-competitive **(consistency)**
  - wrong prediction: $(1 + \frac{1}{\lambda})$-competitive **(robustness)**

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- this is Pareto-optimal (Angelopoulos, Durr, Jin, Kamali, ITCS 2020)

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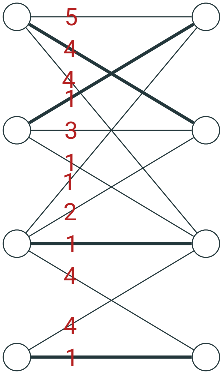

Hungarian algorithm: $O(nm)$ time

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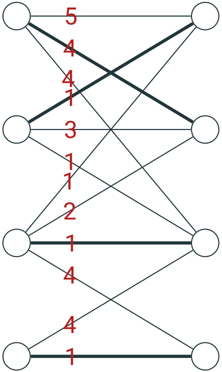

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What if we solve many similar instances? E.g.:

- instances sampled from a distribution,
- one instance slowly changing over time.

RECALL: LP FORMULATION OF MATCHING

Primal:

$$\begin{aligned} \text{minimize} \quad & \sum_{e \in E} c_e x_e \\ \text{subject to} \quad & \sum_{e \in N(v)} x_e = 1 \qquad \forall\, v \in V \\ & x_e \geqslant 0 \qquad \forall\, e \in E \end{aligned}$$

Dual:

$$\begin{aligned} \text{maximize} \quad & \sum_{v \in V} y_v \\ \text{subject to} \quad & y_u + y_v \leqslant c_{u,v} \qquad \forall\, (u, v) \in E \end{aligned}$$

Theorem: (Dinitz et al., NeurIPS 2021)

Suppose input comes with **predicted dual** $\hat{y}$

There is an $O(m\sqrt{n} \cdot \min\{||\hat{y} - y||_1, \sqrt{n}\})$-time algorithm

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Suppose input (edge costs) comes from distribution $\mathcal{D}$, maximum cost $\leqslant C$

Find optimal duals $\hat{y}$ for $\tilde{O}(n^3 C^2)$ samples

W.h.p., $\hat{y}$ approximately (up to $+\varepsilon$) minimizes $\mathbb{E}_{c \sim \mathcal{D}} ||\hat{y} - y_{\mathrm{OPT}}(c)||_1$

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Suppose input (edge costs) comes from distribution $\mathcal{D}$, maximum cost $\leqslant C$

Find optimal duals $\hat{y}$ for $\tilde{O}(n^3C^2)$ samples (i.e., point-wise median of dual solutions)

W.h.p., $\hat{y}$ approximately (up to $+\varepsilon$) minimizes $\mathbb{E}_{c\sim\mathcal{D}}||\hat{y} - y_{\mathrm{OPT}}(c)||_1$

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THANK YOU!